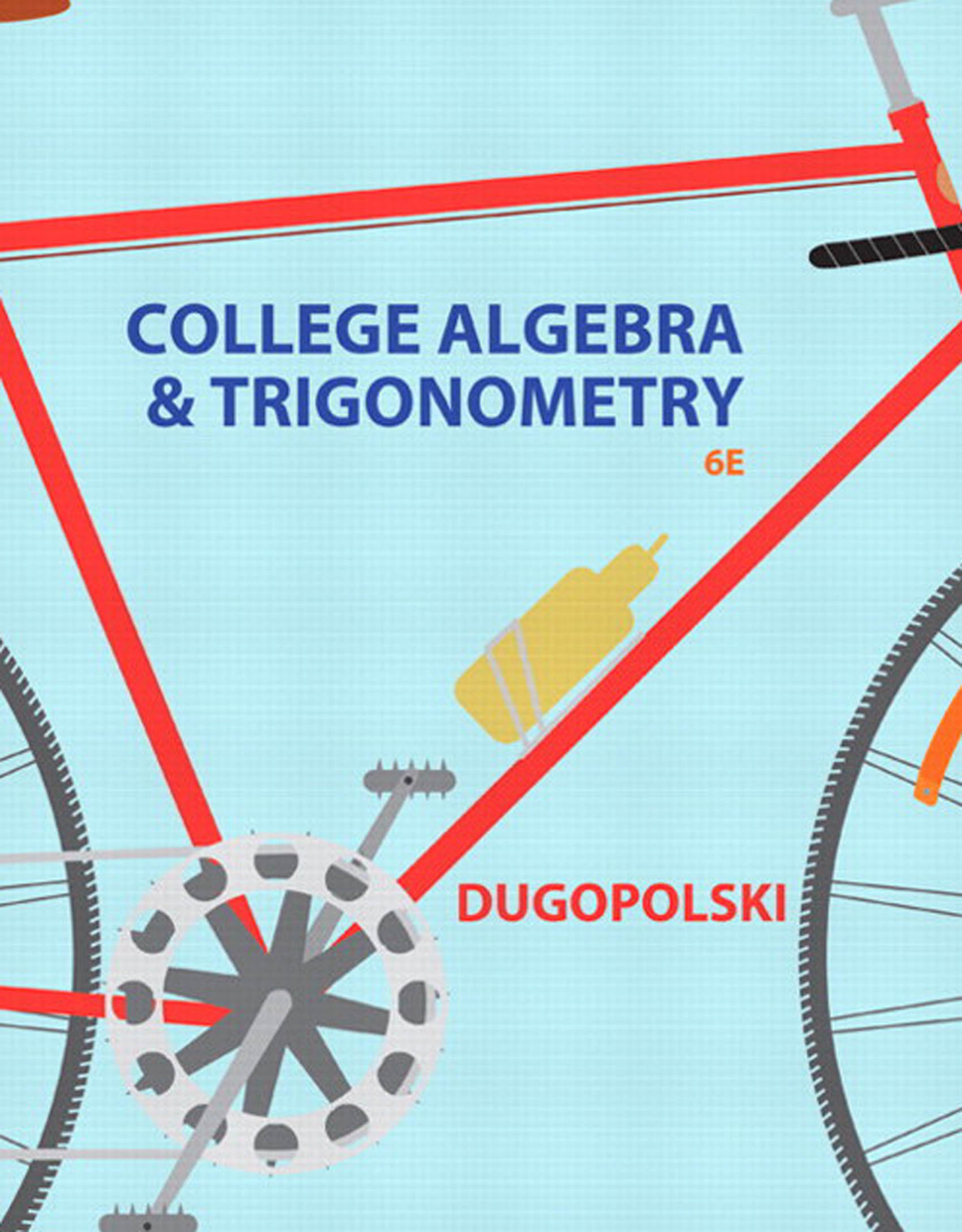
COLLEGE ALGEBRA
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6E
DUGOPOLSKI

Achieve Your Potential

The author, Mark Dugopolski, has developed specific content in MyMathLab® to ensure you have many resources to help you achieve success in mathematics—and beyond! The MyMathLab features described here will help you:

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- **Develop** skills that will help with your transition from college to your profession

Adaptive Study Plan

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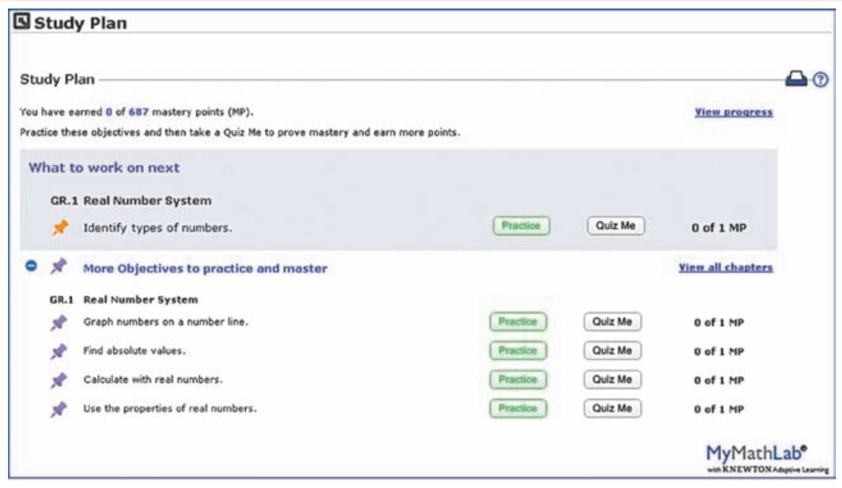
Your performance and activity are assessed continually in real time, providing a personalized experience based on your individual needs.



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The Skills for Success Module supports your continued success in college.

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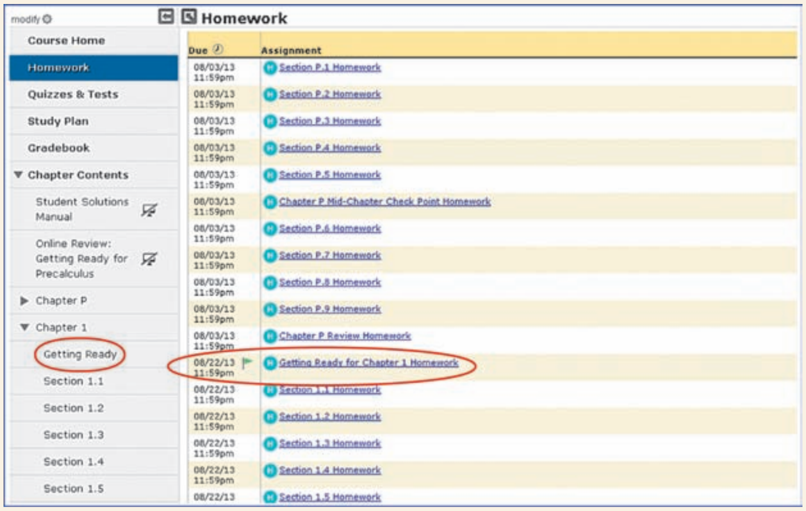

Are you frustrated when you know you learned a math concept in the past, but you can't quite remember the skill when it's time to use it? Don't worry!

The author has included Getting Ready material so you can brush up on forgotten material efficiently by taking a quick skill review quiz to pinpoint the areas where you need help.

Then, a personalized homework assignment provides additional practice on those forgotten concepts, right when you need it.

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College Algebra and Trigonometry 6e

A UNIT CIRCLE APPROACH

Mark Dugopolski
Southeastern Louisiana University

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PREFACE

The sixth edition of *College Algebra and Trigonometry* is designed for a variety of students with different mathematical needs. For those students who will take additional mathematics, this text will provide the proper foundation of skills, understanding, and insights for success in subsequent courses. For those students who will not further pursue mathematics, the extensive emphasis on applications and modeling will demonstrate the usefulness and applicability of algebra and trigonometry in the modern world. I am always on the lookout for real-life applications of mathematics, and I have included many problems that people actually encounter on the job. With an emphasis on problem solving, this text provides students an excellent opportunity to sharpen their reasoning and thinking skills. With increased problem-solving capabilities, students will have confidence to tackle problems that they encounter outside the mathematics classroom.

New to This Edition

For this edition of *College Algebra and Trigonometry*, I have updated explanations, examples, exercises, and art in response to users of the fifth edition. Here are the major changes in this edition:

- Data used in examples, exercises, and chapter openers have been updated to make this text relevant for today's student.
- The number of fill-in-the-blank exercises has been increased to help students master the concepts.
- Section 3.4, Miscellaneous Equations, has been moved to Chapter 1 to give Chapter 3 a better flow and less material.
- Section 1.5, Scatter Diagrams and Curve Fitting, has been moved to Online Appendix A so that Chapter 1 still has seven sections.
- The exercise sets now contain new headings that group the exercises by type: Concepts, Skills, and Modeling.
- Exercise sets now contain QR codes that link the exercises to videos on the Web.
- A new *Video Notebook* gives students a structured place to take notes as they watch the videos. Definitions, examples, and important concepts are highlighted, and helpful hints are pointed out along the way. The notebook is available in MyMathLab®.
- Many new Outside the Box exercises have been added so that there are now two of them at the end of every section.
- Many calculator screen shots have been updated to the TI-84 Plus calculator.
- Suggested homework problems are indicated in the Annotated Instructor's Edition. Each section contains approximately 20 underlined exercise numbers that can be used for an initial assignment of homework exercises.

Continuing Features

With each new edition, all of the features are reviewed to make sure they are providing a positive impact on student success. The continuing features of the text are listed here.

Strategies for Success

- **Chapter Opener** Each chapter begins with chapter opener text that discusses a real-world situation in which the mathematics of the chapter is used. Examples and exercises that relate back to the chapter opener are included in the chapter.

- **Try This** Occurring immediately after every example in the text is an exercise that is very similar to the example. These problems give students the opportunity to immediately check their understanding of the example. Solutions to all *Try This* exercises are in Online Appendix B.
- **Summaries** of important concepts are included to help students clarify ideas that have multiple parts.
- **Strategies** contain general guidelines for solving certain types of problems. They are designed to help students sharpen their problem-solving skills.
- **Procedures** are similar to *Strategies*, but are more specific and more algorithmic. *Procedures* are designed to give students a step-by-step approach for solving a specific type of problem.
- **Function Galleries** Located throughout the text, these function summaries are also gathered together at the end of the text. These graphical summaries are designed to help students link the visual aspects of various families of functions to the properties of the functions.
- **Hints** Selected applications include hints that are designed to encourage students to start thinking about the problem at hand. A *Hint* logo HINT is used where a hint is given.
- **Graphing Calculator Discussions** Optional graphing calculator discussions have been included in the text. They are clearly marked by graphing calculator icons so that they can be easily skipped if desired. Although the graphing calculator discussions are optional, all students will benefit from reading them. In this text, the graphing calculator is used as a tool to support and enhance algebraic conclusions, not to make conclusions.

Section Exercises and Review

- **For Thought** Each exercise set is preceded by a set of ten true/false questions that review the basic concepts in the section, help check student understanding, and offer opportunities for writing and discussion. The answers to all *For Thought* exercises are included in the back of the *Student Edition*.
- **Exercise Sets** The exercise sets range from easy to challenging and are arranged in order of increasing difficulty. Those exercises that require a graphing calculator are optional and are marked with an icon.
- **Writing/Discussion and Cooperative Learning Exercises** These exercises deepen students' understanding by giving them the opportunity to express mathematical ideas both in writing and to their classmates during small group or team discussions.
- **Outside the Box** Found throughout the text, these problems are designed to get students and instructors to do some mathematics just for fun. I enjoyed solving these problems and hope that you will too. The problems can be used for individual or group work. They may or may not have anything to do with the sections in which they are located and might not even require any techniques discussed in the text. So be creative and try thinking *Outside the Box*. The answers are given in the *Annotated Instructor's Edition* only, and complete solutions can be found in the *Instructor's Solutions Manual*.
- **Pop Quizzes** Included at the end of every section of the text, the *Pop Quizzes* give instructors and students convenient quizzes that can be used in the classroom to check understanding of the basics. The answers appear in the *Annotated Instructor's Edition* only.
- **Linking Concepts** This feature is located at the end of nearly every exercise set. It is a multipart exercise or exploration that can be used for individual or group work. The idea of this feature is to use concepts from the current section along with concepts from preceding sections or chapters to solve problems that illustrate the links among various ideas. Some parts of these questions are open-ended and require somewhat more thought than standard skill-building

exercises. Answers are given in the *Annotated Instructor's Edition* only, and full solutions can be found in the *Instructor's Solutions Manual.*

Chapter Review

- **Highlights** This end-of-chapter feature contains an overview of all of the concepts presented in the chapter, along with brief examples to illustrate the concepts.
- **Chapter Review Exercises** These exercises are designed to give students a comprehensive review of the chapter without reference to individual sections and to prepare students for a chapter test.
- **Chapter Test** The problems in the *Chapter Test* are designed to measure the student's readiness for a typical one-hour classroom test. Instructors may also use them as a model for their own end-of-chapter tests. Students should be aware that their in-class test could vary from the *Chapter Test* due to different emphasis placed on the topics by individual instructors.
- **Tying It All Together** Found at the end of most chapters in the text, these exercises help students review selected concepts from the present and prior chapters and require students to integrate multiple concepts and skills.
- **Index of Applications** The many applications contained within the text are listed in the *Index of Applications* that appears at the end of the text. The applications are page referenced and grouped by subject matter.

RESOURCES FOR SUCCESS

MyMathLab® Online Course (access code required)

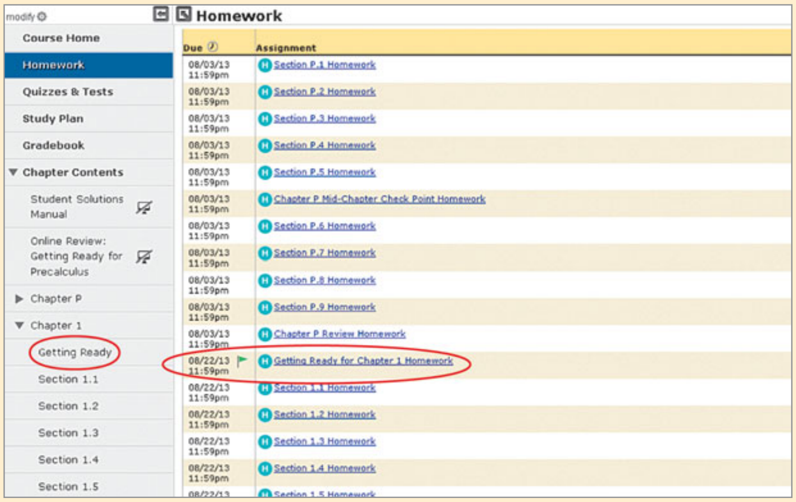
MyMathLab from Pearson is the world's leading online resource in mathematics, integrating interactive homework, assessment, and media in a flexible, easy-to-use format. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And it comes from an **experienced partner** with educational expertise and an eye on the future.

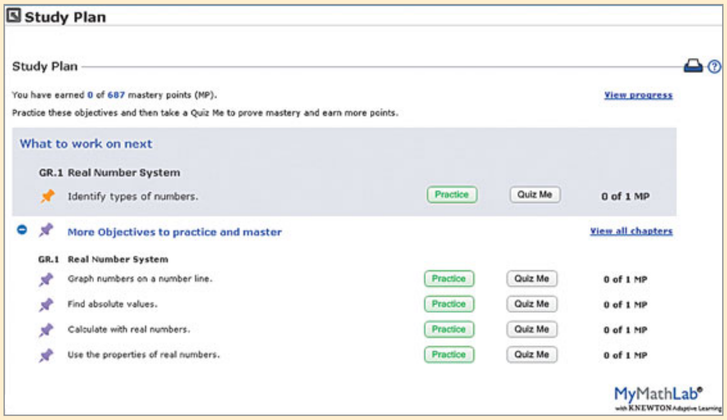
To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit **www.mymathlab.com** or contact your Pearson representative.

MyMathLab helps prepare students and gets them thinking more conceptually and visually through the following features:

Getting Ready

Students refresh prerequisite topics through assignable skill review quizzes and personalized homework integrated in MyMathLab.





Adaptive Study Plan

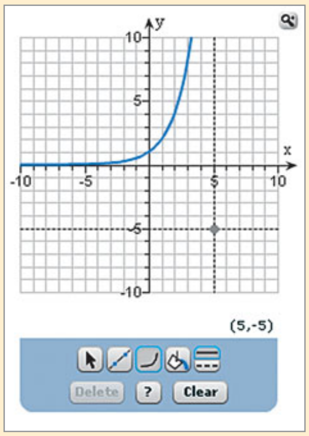
The Study Plan makes studying more efficient and effective for every student. Performance and activity are assessed continually in real time. The data and analytics are used to provide personalized content, reinforcing concepts that target each student's strengths and weaknesses.

Skills for Success Modules

Integrated within MyMathLab, these modules help students succeed in collegiate courses and prepare for future professions.

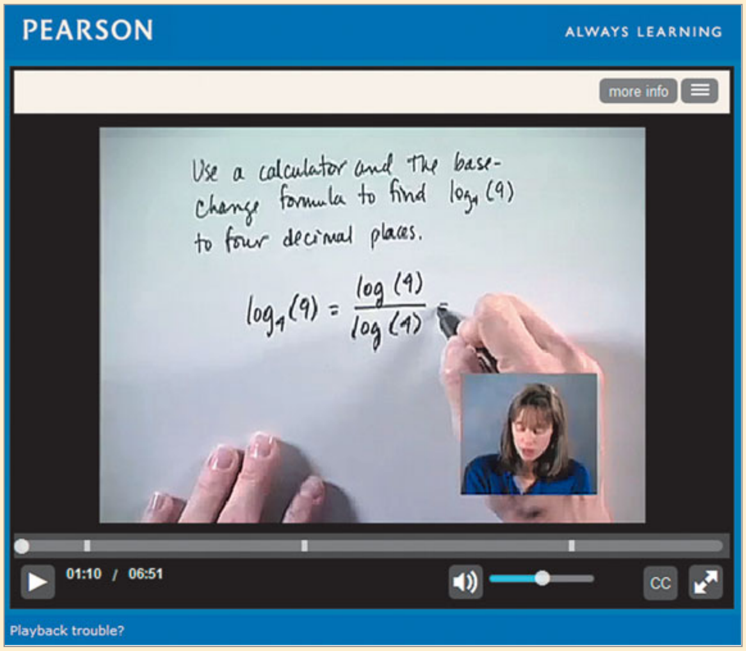
Review Your Knowledge

Review Exercises in the text are now assignable in MyMathLab and require students to recall previously learned content and skills. By helping students maintain essential skills throughout the course, they retain information that prepares them for future math courses.



Enhanced Graphing Functionality

New functionality within the graphing utility allows graphing of 3-point quadratic functions, 4-point cubic graphs, and transformations in exercises.



Video Assessment

Video assessment is tied to the video lecture for each section of the book to check students' understanding of important math concepts. Instructors can assign these questions as a prerequisite to homework assignments.

Instructor Resources

Additional resources can be downloaded from www.pearsonhighered.com or hardcopy resources can be ordered from your sales representative.

Ready to Go Courses

Now it is even easier to get started with MyMathLab. The Ready to Go MyMathLab course option includes author-chosen preassigned homework, integrated review, and more.

TestGen®

TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

PowerPoint® Lecture Slides

Classroom presentation slides are geared specifically to sequence the text. Available in MyMathLab.

Annotated Instructor's Edition

Provides answers beside the text exercises for most exercises and in an answer section at the back of the book for all others. Groups of exercises are keyed back to corresponding examples from the section.

Instructor's Solutions Manual

Provides complete solutions to all text exercises, including the *For Thought* and *Linking Concepts* exercises.

Student Resources

Additional resources are available to help students succeed.

Lecture Video

Example and content videos provide comprehensive coverage of each section and topic in the text in an engaging format that stresses student interaction. Includes optional subtitles in English and Spanish. All videos are assignable within MyMathLab.

Student's Solutions Manual

Provides detailed solutions to odd-numbered exercises in the text.

NEW Video Notebook

This printed guide gives students a structured place to take notes as they watch the videos. Definitions, examples, and important concepts are highlighted, and helpful hints are pointed out along the way. The notebook is available in MyMathLab.

Graphing Calculator Manual

Provides instructions and keystroke operations for the TI-83/84 Plus and TI-89. Contains worked-out examples taken directly from the text. Available in MyMathLab.

ACKNOWLEDGMENTS

Thanks to the many professors and students who have used this text in previous editions. I am always glad to hear from users of my texts. You can e-mail me at bookinit@charter.net. Thanks to the following reviewers, whose comments and suggestions were invaluable in preparing this sixth edition:

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As always, thanks to my wife, Cheryl, whose love, encouragement, understanding, support, and patience are invaluable.

Mark Dugopolski
Ponchatoula, Louisiana

P Prerequisites

Gone are the days when raw sailing ability and stamina won races like the America's Cup. In the modern sailing world, technology plays an ever increasing role in determining outcomes. All yachts now have a crewman on board to interpret computer data on sailing conditions.

Today mathematical formulas are used in designing the boats and in stating the rules that govern the race. Formulas are used to keep the race competitive by establishing strict boundaries for yacht sail area, hull shape, length, and displacement. Yachtsmen must do their homework and prepare for this race for years in advance. Preparation is just as important in algebra.

WHAT YOU WILL **LEARN**

In this chapter we will review the basic concepts that are necessary for success in algebra. Throughout this chapter you will see that even basic concepts have applications in business, science, engineering, and sailing.

P.1 Real Numbers and Their Properties

In arithmetic we learn facts about the real numbers and how to perform operations with them. Since algebra is an extension of arithmetic, we begin our study of algebra with a discussion of the real numbers and their properties.

The Real Numbers

A **set** is a collection of objects or **elements.** The set containing the numbers 1, 2, and 3 is written as $\{1, 2, 3\}$. To indicate a continuing pattern, we use three dots as in $\{1, 2, 3, \ldots\}$. The set of real numbers is a collection of many types of numbers. To better understand the real numbers we recall some of the basic subsets of the real numbers:

Subset	Name (symbol)
$\{1, 2, 3, \ldots\}$	**Counting** or **natural numbers** (N)
$\{0, 1, 2, 3, \ldots\}$	**Whole numbers** (W)
$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$	**Integers** (Z)

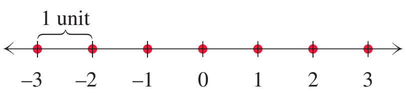

Figure P.1

Numbers can be pictured as points on a line, the **number line.** To draw a number line, draw a line and label any convenient point with the number 0. Now choose a convenient length, one **unit,** and use it to locate evenly spaced points as shown in Fig. P.1. The positive integers are located to the right of zero and the negative integers to the left of zero. The numbers corresponding to the points on the line are called the **coordinates** of the points.

The integers and their ratios form the set of **rational numbers,** Q. The rational numbers also correspond to points on the number line. For example, the rational number $1/2$ is found halfway between 0 and 1 on the number line. In set notation, the set of rational numbers is written as

$$\left\{\frac{a}{b}\,\middle|\, a \text{ and } b \text{ are integers with } b \neq 0\right\}.$$

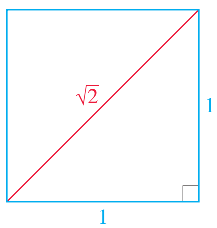

Figure P.2

This notation is read "The set of all numbers of the form a/b such that a and b are integers with b not equal to zero." In our set notation we used letters to represent integers. A letter that is used to represent a number is called a **variable.**

There are infinitely many rational numbers located between each pair of consecutive integers, yet there are infinitely many points on the number line that do not correspond to rational numbers. The numbers that correspond to those points are called **irrational** numbers. In decimal notation, the rational numbers are the numbers that are repeating or terminating decimals, and the irrational numbers are the nonrepeating nonterminating decimals. For example, the number $0.595959\ldots$ is a rational number because the pair 59 repeats indefinitely. By contrast, notice that in the number $5.010010001\ldots$, each group of zeros contains one more zero than the previous group. Because no group of digits repeats, $5.010010001\ldots$ is an irrational number.

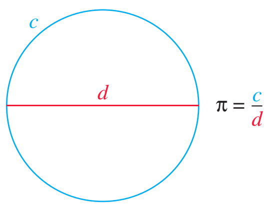

Figure P.3

Numbers such as $\sqrt{2}$ and π are also irrational. We can visualize $\sqrt{2}$ as the length of the diagonal of a square whose sides are one unit in length. See Fig. P.2. In any circle, the ratio of the circumference c to the diameter d is π $(\pi = c/d)$. See Fig. P.3. It is difficult to see that numbers like $\sqrt{2}$ and π are irrational because their decimal representations are not apparent. However, the irrationality of π was proven in 1767 by Johann Heinrich Lambert, and it can be shown that the square root of any positive integer that is not a perfect square is irrational.

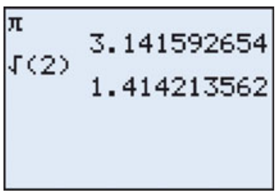

Figure P.4

Since a calculator operates with a fixed number of decimal places, it gives us a *rational approximation* for an irrational number such as $\sqrt{2}$ or π. See Fig. P.4. □

The set of rational numbers, Q, together with the set of irrational numbers, I, is called the set of **real numbers,** R. The following are examples of real numbers:

$$-3, \quad -0.025, \quad 0, \quad \frac{1}{3}, \quad 0.595959\ldots, \quad \sqrt{2}, \quad \pi, \quad 5.010010001\ldots$$

These numbers are **graphed** on a number line in Fig. P.5.

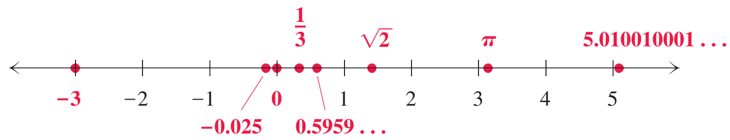

Figure P.5

Since there is a one-to-one correspondence between the points of the number line and the real numbers, we often refer to a real number as a point. Figure P.6 shows how the various subsets of the real numbers are related to one another.

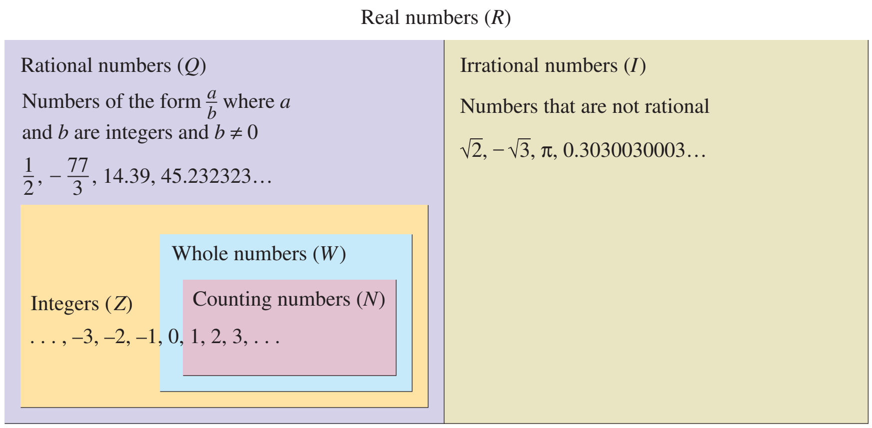

Figure P.6

To indicate that a number is a member of a set, we write $a \in A$, which is read "a is a member of set A." We write $a \notin A$ for "a is not a member of set A." Set A is a subset of set B $(A \subseteq B)$ means that every member of set A is also a member of set B, and A is not a subset of B $(A \not\subseteq B)$ means that there is at least one member of A that is not a member of B.

EXAMPLE 1 Classifying numbers and sets of numbers

Determine whether each statement is true or false and explain. See Fig. P.6.

a. $0 \in R$ **b.** $\pi \in Q$ **c.** $R \subseteq Q$ **d.** $I \not\subseteq Q$ **e.** $\sqrt{5} \in Q$

Solution

a. True, because 0 is a member of the set of whole numbers, which is a subset of the set of real numbers.
b. False, because π is irrational.
c. False, because every irrational number is a member of R but not Q.
d. True, because the irrational numbers and the rational numbers have no numbers in common.
e. False, because the square root of any integer that is not a perfect square is irrational.

TRY THIS. True or false? **a.** $0 \in I$ **b.** $I \subseteq R$ **c.** $\sqrt{5} \in I$ ■

Properties of the Real Numbers

In arithmetic we can observe that $3 + 4 = 4 + 3$, $6 + 9 = 9 + 6$, etc. We get the same sum when we add two real numbers in either order. This property of addition of real numbers is the **commutative property.** Using variables, the commutative property of addition is stated as $a + b = b + a$ for any real numbers a and b. There is also a commutative property of multiplication, which is written as $a \cdot b = b \cdot a$ or $ab = ba$. There are many properties concerning the operations of addition and multiplication on the real numbers that are useful in algebra.

Properties of the Real Numbers

For any real numbers a, b, and c:

1. $a + b$ and ab are real numbers.	**Closure property**
2. $a + b = b + a$ and $ab = ba$	**Commutative properties**
3. $a + (b + c) = (a + b) + c$ and $a(bc) = (ab)c$	**Associative properties**
4. $a(b + c) = ab + ac$	**Distributive property**
5. $0 + a = a$ and $1 \cdot a = a$ (Zero is the **additive identity,** and 1 is the **multiplicative identity.**)	**Identity properties**
6. $0 \cdot a = 0$	**Multiplication property of zero**
7. For each real number a, there is a unique real number $-a$ such that $a + (-a) = 0$. ($-a$ is the **additive inverse** of a.)	**Additive inverse property**
8. For each nonzero real number a, there is a unique real number $1/a$ such that $a \cdot 1/a = 1$. ($1/a$ is the **multiplicative inverse** or **reciprocal** of a.)	**Multiplicative inverse property**

The closure property indicates that the sum and product of any pair of real numbers is a real number. The commutative properties indicate that we can add or multiply in either order and get the same result. Since we can add or multiply only a pair of numbers at one time, the associative properties indicate two different ways to obtain the same result when adding or multiplying three numbers. The operations within parentheses are performed first. Because of the commutative property, the distributive property can be used also in the form $(b + c)a = ab + ac$.

Note that the properties stated here involve only addition and multiplication, considered the basic operations of the real numbers. Subtraction and division are defined in terms of addition and multiplication. By definition $a - b = a + (-b)$ and $a \div b = a \cdot 1/b$ for $b \neq 0$. Note that $a - b$ is called the **difference** of a and b and $a \div b$ is called the **quotient** of a and b.

EXAMPLE 2 Using the properties

Complete each statement using the property named.

a. $a7 =$ __________, commutative
b. $2x + 4 =$ __________, distributive
c. $8($__________$) = 1$, multiplicative inverse
d. $\frac{1}{3}(3x) =$ __________, associative

Solution

a. $a7 = 7a$ **b.** $2x + 4 = 2(x + 2)$

c. $8\left(\frac{1}{8}\right) = 1$ **d.** $\frac{1}{3}(3x) = \left(\frac{1}{3} \cdot 3\right)x$

TRY THIS. Complete the statement $x \cdot 3 =$ ________ using the commutative property. ■

Additive Inverses

The negative sign is used to indicate negative numbers as in -7 (negative seven). If the negative sign precedes a variable as in $-b$ it is read as "additive inverse" or "opposite" because $-b$ could be positive or negative. If b is positive then $-b$ is negative, and if b is negative then $-b$ is positive.

Using two "opposite" signs has a cancellation effect. For example, $-(-5) = 5$ and $-(-(-3)) = -3$. Note that the additive inverse of a number can be obtained by multiplying the number by -1. For example, $-1 \cdot 3 = -3$.

Calculators usually use the sign (-) to indicate opposite or negative and the subtraction sign $(-)$ for subtraction as shown in Fig. P.7. □

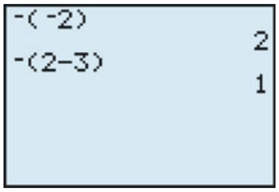

Figure P.7

We know that $a + b = b + a$ for any real numbers a and b, but is $a - b = b - a$ for any real numbers a and b? In general, $a - b$ is not equal to $b - a$. For example, $7 - 3 = 4$ and $3 - 7 = -4$. So subtraction is not commutative. Since $a - b + b - a = 0$, we can conclude that $a - b$ and $b - a$ are opposites or additive inverses of each other. We summarize these properties of opposites as follows.

Properties of Opposites

For any real numbers a and b:

1. $-1 \cdot a = -a$ (The product of -1 and a is the opposite of a.)
2. $-(-a) = a$ (The opposite of the opposite of a is a.)
3. $-(a - b) = b - a$ (The opposite of $a - b$ is $b - a$.)

EXAMPLE 3 Using properties of opposites

Use the properties of opposites to complete each equation.

a. $-(-\pi) =$ ______ **b.** $-1(-2) =$ ______ **c.** $-1(x - h) =$ ______

Solution

a. $-(-\pi) = \pi$
b. $-1(-2) = -(-2) = 2$
c. $-1(x - h) = -(x - h) = h - x$

TRY THIS. Complete the equation $-(1 - w) =$ ______ using the properties of opposites. ■

Relations

Symbols such as $<, >, =, \leq$, and $\geq$ are called **relations** because they indicate how numbers are related. We can visualize these relations by using a number line. For example, $\sqrt{2}$ is located to the right of 0 in Fig. P.5, so $\sqrt{2} > 0$. Since $\sqrt{2}$ is to the left of π in Fig. P.5, $\sqrt{2} < \pi$. In fact, if a and b are any two real numbers, we say that a is less than b (written $a < b$) provided that a is to the left of b on the number line. We say that a is greater than b (written $a > b$) if a is to the right of b

on the number line. We say $a = b$ if a and b correspond to the same point on the number line. The fact that there are only three possibilities for ordering a pair of real numbers is called the **trichotomy property.**

Trichotomy Property

For any two real numbers a and b, exactly one of the following is true: $a < b, a > b$, or $a = b$.

The trichotomy property is very natural to use. For example, if we know that $r = t$ is false, then we can conclude (using the trichotomy property) that either $r < t$ or $r > t$ is true. If we know that $w + 6 > z$ is false, then we can conclude that $w + 6 \leq z$ is true. The following four properties of equality are also very natural to use, and we often use them without even thinking about them.

Properties of Equality

For any real numbers a, b, and c:

1. $a = a$ — **Reflexive property**
2. If $a = b$, then $b = a$. — **Symmetric property**
3. If $a = b$ and $b = c$, then $a = c$. — **Transitive property**
4. If $a = b$, then a and b may be substituted for one another in any expression involving a or b. — **Substitution property**

Absolute Value

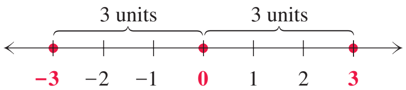

Figure P.8

The **absolute value** of a (in symbols, $|a|$) can be thought of as the distance from a to 0 on a number line. Since both 3 and -3 are three units from 0 on a number line as shown in Fig. P.8, $|3| = 3$ and $|-3| = 3$.

A symbolic definition of absolute value is written as follows.

Definition: Absolute Value

For any real number a,

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0. \end{cases}$$

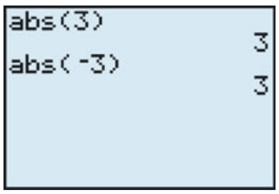

Figure P.9

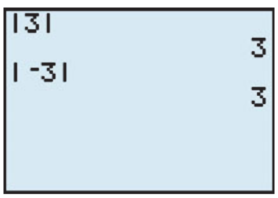

Figure P.10

A TI-84 Plus calculator can find absolute value in classic mode as in Fig. P.9 or in MathPrint mode as in Fig. P.10. □

The symbolic definition of absolute value indicates that for $a \geq 0$ we use the equation $|a| = a$ (the absolute value of a is just a). For $a < 0$ we use the equation $|a| = -a$ (the absolute value of a is the opposite of a, a positive number).

EXAMPLE 4 Using the definition of absolute value

Use the symbolic definition of absolute value to simplify each expression.

a. $|5.6|$ **b.** $|0|$ **c.** $|-3|$

Solution

a. Since $5.6 \geq 0$, we use the equation $|a| = a$ to get $|5.6| = 5.6$.
b. Since $0 \geq 0$, we use the equation $|a| = a$ to get $|0| = 0$.
c. Since $-3 < 0$, we use the equation $|a| = -a$ to get $|-3| = -(-3) = 3$.

TRY THIS. Use the definition of absolute value to simplify $|-9|$. ■

The definition of absolute value guarantees that the absolute value of any number is nonnegative. The definition also implies that additive inverses (or opposites) have the same absolute value. These properties of absolute value and two others are stated as follows.

Properties of Absolute Value

For any real numbers a and b:

1. $|a| \geq 0$ (The absolute value of any number is nonnegative.)
2. $|-a| = |a|$ (Additive inverses have the same absolute value.)
3. $|a \cdot b| = |a| \cdot |b|$ (The absolute value of a product is the product of the absolute values.)
4. $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, b \neq 0$ (The absolute value of a quotient is the quotient of the absolute values.)

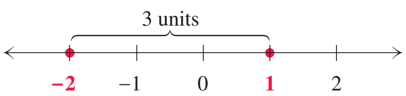

Figure P.11

Absolute value is used in finding the distance between points on a number line. Since 9 lies four units to the right of 5, the distance between 5 and 9 is 4. In symbols, $d(5, 9) = 4$. We can obtain 4 by $9 - 5 = 4$ or $|5 - 9| = 4$. In general, $|a - b|$ gives the distance between a and b for any values of a and b. For example, the distance between -2 and 1 in Fig. P.11 is three units and

$$d(-2, 1) = |-2 - 1| = |-3| = 3.$$

Distance between Two Points on the Number Line

If a and b are any two points on the number line, then the distance between a and b is $|a - b|$. In symbols, $d(a, b) = |a - b|$.

Note that $d(a, 0) = |a - 0| = |a|$, which is consistent with the definition of absolute value of a as the distance between a and 0 on the number line.

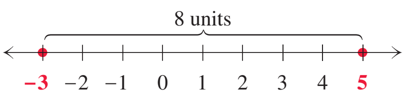

Figure P.12

EXAMPLE 5 Distance between two points on a number line

Find the distance between -3 and 5 on the number line.

Solution

The points corresponding to -3 and 5 are shown on the number line in Fig. P.12. The distance between these points is found as follows:

$$d(-3, 5) = |-3 - 5| = |-8| = 8$$

Notice that $d(-3, 5) = d(5, -3)$:

$$d(5, -3) = |5 - (-3)| = |8| = 8$$

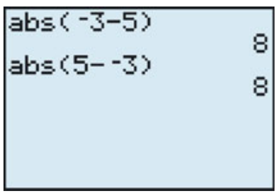

Figure P.13

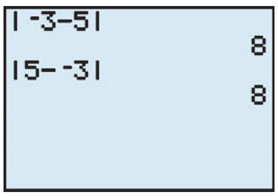

Figure P.14

A TI-84 Plus finds absolute value of a difference in a classic mode as in Fig. P.13 or in MathPrint mode as in Fig. P.14.

TRY THIS. Find the distance between -5 and -9 on the number line. ■

Exponential Expressions

We use positive integral exponents to indicate the number of times a number occurs in a product. For example, $2 \cdot 2 \cdot 2 \cdot 2$ is written as 2^4. We read 2^4 as "the fourth power of 2" or "2 to the fourth power."

Definition: Positive Integral Exponents

For any positive integer n

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n \text{ factors of } a}.$$

We call a the **base,** n the **exponent** or **power,** and a^n an **exponential expression.**

We read a^n as "a to the nth power." For a^1 we usually omit the exponent and write a. We refer to the exponents 2 and 3 as squares and cubes. For example, 3^2 is read "3 squared," 2^3 is read "2 cubed," x^4 is read "x to the fourth," b^5 is read "b to the fifth," and so on. To evaluate an expression such as -3^2 we square 3 first, then take the opposite. So $-3^2 = -9$ and $(-3)^2 = (-3)(-3) = 9$.

EXAMPLE 6 Evaluating exponential expressions

Evaluate.

a. 4^3 **b.** $(-2)^4$ **c.** -2^4

Solution

a. $4^3 = 4 \cdot 4 \cdot 4 = 16 \cdot 4 = 64$
b. $(-2)^4 = (-2)(-2)(-2)(-2) = 16$
c. $-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2) = -16$

TRY THIS. Evaluate. **a.** 5^2 **b.** -5^2 ■

Arithmetic Expressions

The result of writing numbers in a meaningful combination with the ordinary operations of arithmetic is called an **arithmetic expression** or simply an **expression.** The **value** of an arithmetic expression is the real number obtained when all operations are performed. Symbols such as parentheses, brackets, braces, absolute value bars, and fraction bars are called **grouping symbols.** Operations within grouping symbols are performed first.

EXAMPLE 7 Evaluating an arithmetic expression with grouping symbols

Evaluate each expression.

a. $(-7 \cdot 3) + (5 \cdot 8)$ **b.** $\dfrac{3 - 9}{-2 - (-5)}$ **c.** $3 - |5 - (2 \cdot 9)|$

Solution

a. Perform the operations within the parentheses first and remove the parentheses:

$$(-7 \cdot 3) + (5 \cdot 8) = -21 + 40 = 19$$

b. Since the fraction bar acts as a grouping symbol, we evaluate the numerator and denominator before dividing.

$$\frac{3 - 9}{-2 - (-5)} = \frac{-6}{3} = -2$$

c. First evaluate within the innermost grouping symbols:

$$\begin{aligned} 3 - |5 - (2 \cdot 9)| &= 3 - |5 - 18| && \text{Innermost grouping symbols} \\ &= 3 - |-13| && \text{Innermost grouping symbols} \\ &= 3 - 13 && \text{Evaluate the absolute value.} \\ &= -10 && \text{Subtract.} \end{aligned}$$

TRY THIS. Evaluate. **a.** $(-1 + 3)(5 - 6)$ **b.** $2 - |3 - 9|$ ■

The Order of Operations

When some or all grouping symbols are omitted in an expression, we evaluate the expression using the following order of operations.

Order of Operations

1. Evaluate exponential expressions.
2. Perform multiplication and division in order from left to right.
3. Perform addition and subtraction in order from left to right.

Any operations contained within grouping symbols are performed first, using the above order.

EXAMPLE 8 Using the order of operations to evaluate an expression

Evaluate each expression.

a. $3 - 4 \cdot 2^3$ **b.** $5 \cdot 8 \div 4 \cdot 2$ **c.** $3 - 4 + 9 - 2$ **d.** $5 - 2(3 - 4 \cdot 2)^2$

Solution

a. By the order of operations evaluate 2^3, then multiply, and then subtract:

$$3 - 4 \cdot 2^3 = 3 - 4 \cdot 8 = 3 - 32 = -29$$

b. In an expression with only multiplication and division, the operations are performed from left to right:

$$5 \cdot 8 \div 4 \cdot 2 = 40 \div 4 \cdot 2 = 10 \cdot 2 = 20$$

c. In an expression with only addition and subtraction, the operations are performed from left to right:

$$3 - 4 + 9 - 2 = -1 + 9 - 2 = 8 - 2 = 6$$

d. Perform operations within parentheses first, using the order of operations:

$$5 - 2(3 - 4 \cdot 2)^2 = 5 - 2(-5)^2 = 5 - 2 \cdot 25 = -45$$

TRY THIS. Evaluate. **a.** $3 - 6 \cdot 2$ **b.** $4 - 5 \cdot 2^3$ ■

Algebraic Expressions

When we write numbers and one or more variables in a meaningful combination with the ordinary operations of arithmetic, the result is called an **algebraic expression,** or simply an expression. The **value of an algebraic expression** is the value of the arithmetic expression that is obtained when the variables are replaced by real numbers.